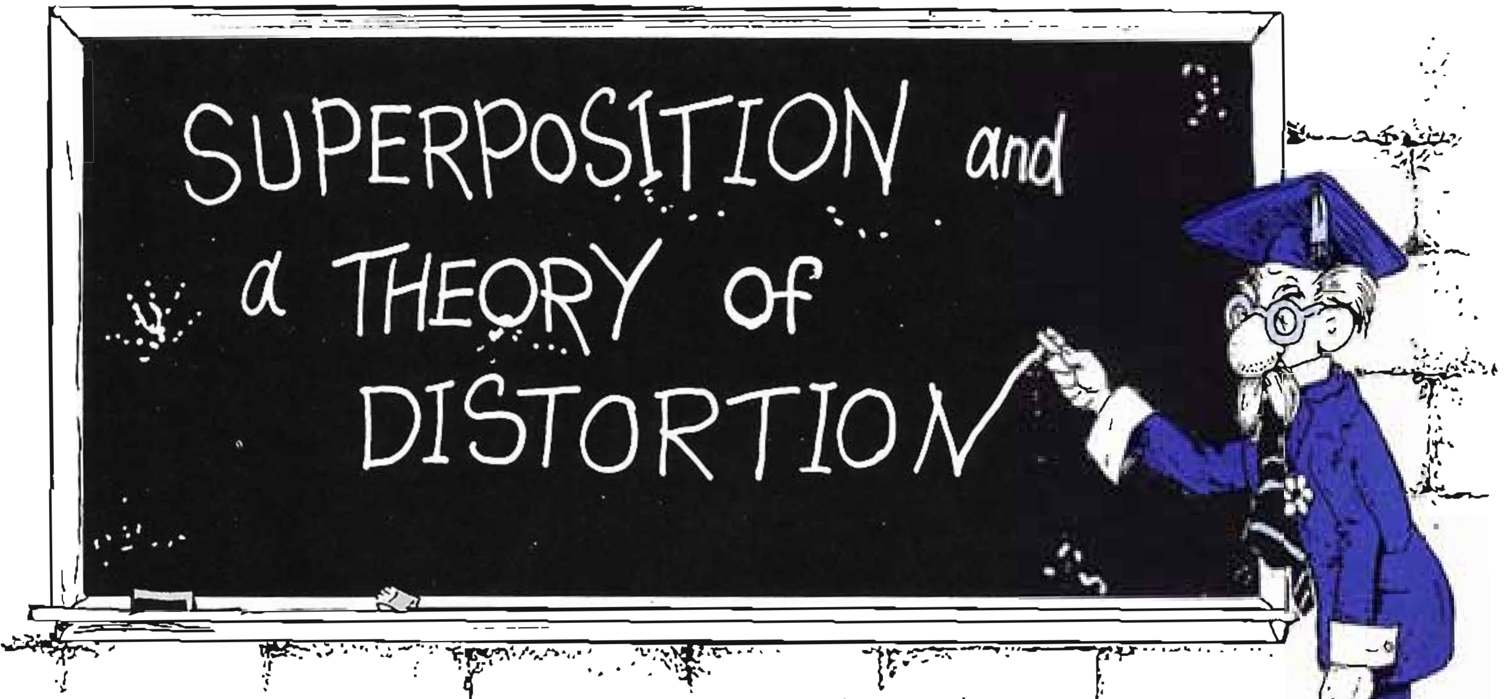
SUPERPOSITION and a THEORY of DISTORTION



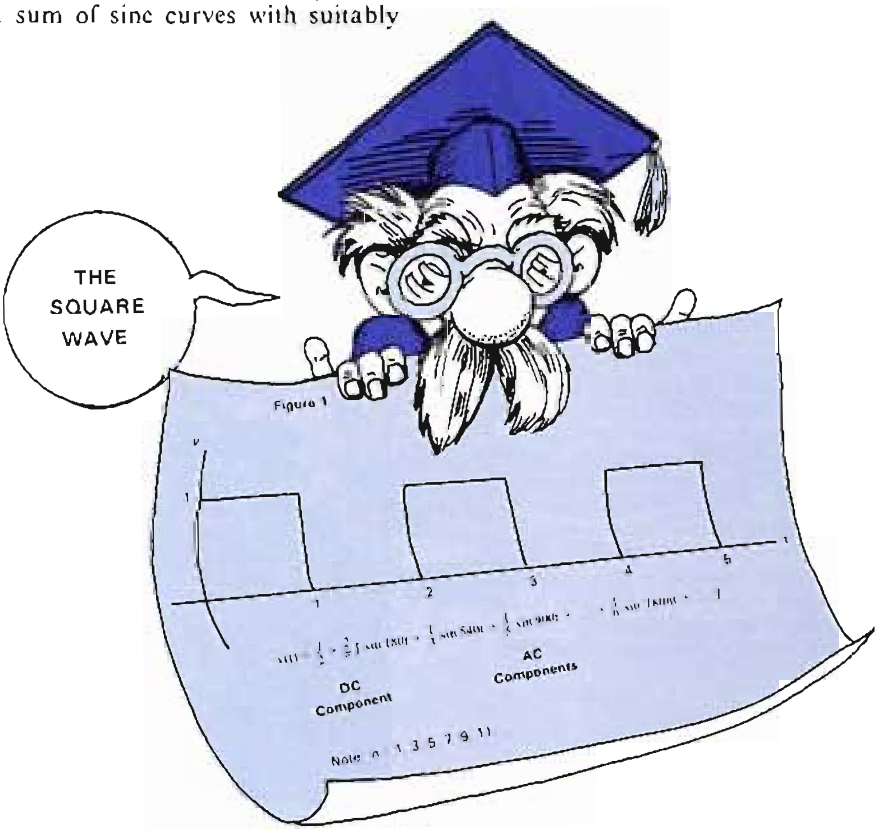
by Dr. Brian Kelly

The principle of superposition is well known to communications technicians. The principle states that the net effect on a circuit by several voltage sources is the algebraic sum of the effects produced independently by each source. Further, each single voltage source may itself be treated as the composite of independent signals of various frequencies.

In mathematical terms, the principle states that a waveform may be expressed as a sum of sine curves with suitably chosen amplitudes. This expression, which is called a Fourier Series, is essentially unique, and has been a standard technique in engineering applications for more than 100 years.

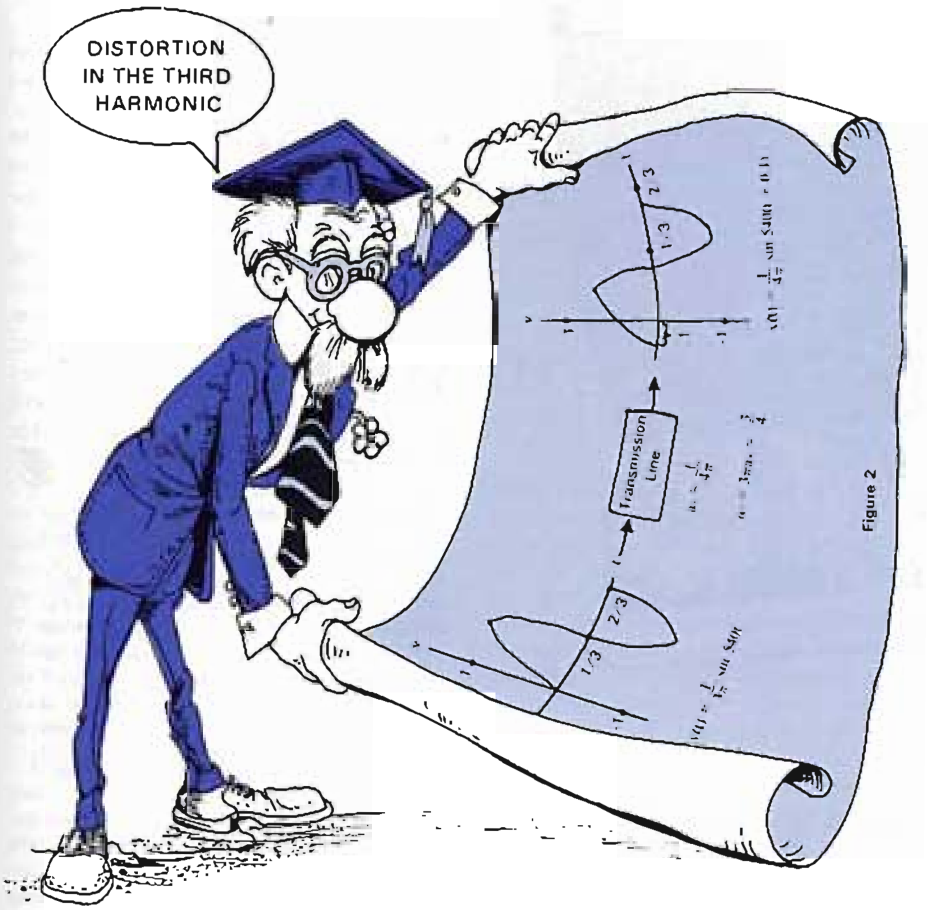
The principle of superposition is flexible in its application. Looking at the principle one way, several independent signals, each with its own distinct frequency or frequency band, may be combined or "stacked" into a single composite signal. Thus, superposition is fundamental to such applications as Frequency Division Multiplexing (FDM) in carrier systems.

Looking at the same principle another way, superposition allows the effect of a transmission line on a given signal to be determined by examining how the line affects the component signals. This effect, called distortion, can be analyzed independently at selected frequencies. Adopting this point of view, we can put forth a simple theory of distortion based on the principle of superposition: "The output signal will be the algebraic sum of the distorted component signals."



Take, for example, the square wave. This waveform is generated in manual teletype systems by the standard R-Y test signal and is the basic wave form for both data and Pulse Code Modulation (PCM) transmissions. The square wave's graphical simplicity hides the fact that it may be viewed as a composite of an infinite number of sine waves, each of a different frequency.

In figure 1 the square wave is represented as a DC component plus



AC components. The latter consists of all the odd harmonics of a basic sine wave. The phase angles are taken in degrees and the amplitudes and frequencies have been normalized to convenient units. The summation is exact except at the breaks ($t = 0, \pm 1, \pm 2, \ldots$) where the series sums to ½.

The factor $2/(n\pi)$ is called the coefficient of the n^{th} term of the series. n takes on only odd values since the series consists of only the odd harmonics of the basic sine wave. Note that the higher the harmonic the less is its contribution to the composite signal, since the coefficients become successively smaller in magnitude with increasing frequency.

To describe the effect of a transmission line on this signal, it is customary to set up an appropriate mathematical model. As was stated earlier, according to the principle of superposition, the total effect of the line on a signal may be determined from the effect of the line on the various component signals. So, let us now examine a typical component.

The n^{th} harmonic component of the square wave is

$$(2/n\pi)\sin 180\,nt$$

If this signal alone is passed through a circuit, it will come out in the form

$$a_n \sin(180n(t - \delta_n)).$$

The ratio $\alpha_n = a_n/(2/n\pi) = \alpha n\pi a_n/2$ is called the amplification factor. If $\alpha_n > 1$ (that is, if $a_n > 2/(n\pi)$, the circuit is said to amplify the signal. If $\alpha_n < 1$ (that is, if $a_n < 2/(n\pi)$), the circuit is said to attenuate the signal. α_n has the effect of delaying or shifting the signal; hence, it measures the frequency shift distortion. Since attenuation and phase shift are frequency dependent phenomena, the alphas and deltas are indexed according to their respective harmonics.

Figure 2 depicts a hypothetical example of distortion in the third harmonic ($n = 3$) of the square wave. In this example, $a_3 = 1/(4\pi)$ and $\alpha_3 = 0.1$. Thus $\alpha_3 = 3/4$. If the third harmonic were not distorted, then $\alpha_3 = 1$ and $\alpha_3 = 0$.

Practical transmission systems will pass only certain frequencies. Signals with component frequencies outside these limits may be thought of as having these frequencies completely attenuated. In our model, the coefficients of the terms of the Fourier Series corresponding to these frequencies in the output signal become negligible or even zero. Figure 3 depicts an idealized transmission line which passes the first three harmonics of the square wave without distortion ($\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 0$); but completely attenuates the higher frequencies ($0 = \alpha_4 = \alpha_5 = \ldots$ and $0 = \delta_4 = \delta_5 = \ldots$).

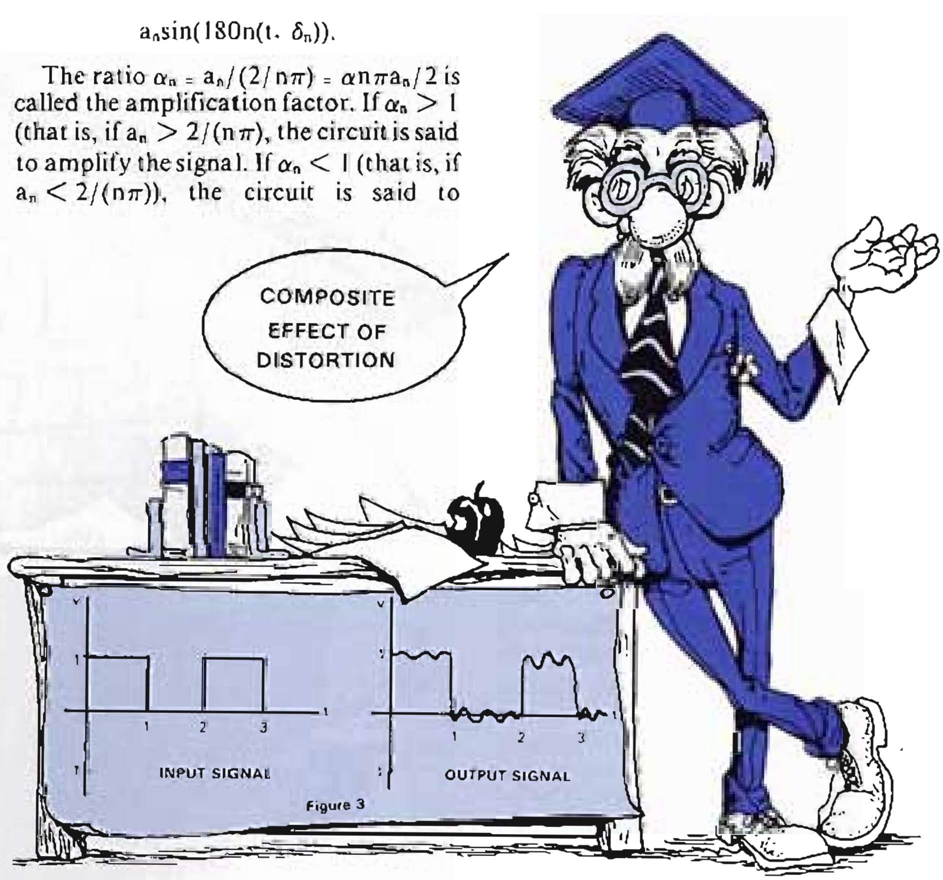


Figure 3

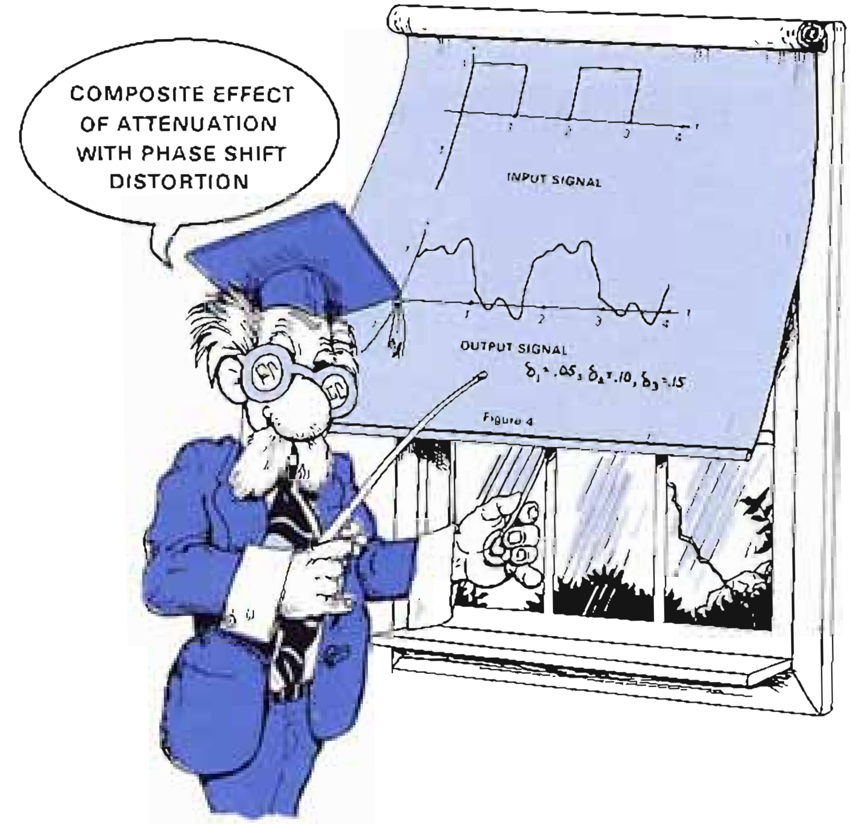


Figure 4

In the absence of other types of distortion, such as random white noise, the output signal in figure 3 would fall within 5 per cent of the original signal for most of the duration of the pulse. If, however, a small phase shift is introduced, say $\delta_1 = 0.05$, $\delta_2 = 0.1$, and $\delta_3 = 0.15$, the output signal will vary as much as 25 per cent from the input signal even during the middle third of the pulse, as is shown in figure 4. Should the phase shift distortion increase even more, say $\delta_1 = 0.1$, $\delta_3 = 0.2$, $\delta_5 = 0.3$, this error exceeds 34 per cent of the original signal. See figure 5.

A transmission system will function effectively only if the effect of distortion from all sources falls within acceptable limits. These limits are set primarily by the type and quality of service required, by the specific equipment employed, and, of course, by the skill of the operators. The successively deteriorating output signals of figures 3, 4 and 5, for example, could not be restored to their original shape by simple amplification. Repeater and relay stations do not eliminate distortion but, rather, are designed to maintain signal quality within specific limits. The compound effect of distortion, however slight at each stage, will then place practical limits on the effective range of a cable or relay system.

In this brief article, we have used the principle of superposition to explain two major types of distortion: attenuation and phase shift. Both are frequency dependent: that is, they may be different for different frequencies.

The Fourier Series of a signal identifies its component frequencies; hence, by examining the Fourier Series of both the input and output signals, we may quantify the distortion. Alternately, given the response characteristics of a circuit and the Fourier Series of the input signal, the output signal can be predicted.

The square wave is well suited to this type of analysis since its component frequencies span such a broad spectrum while its Fourier Series remains rather simple. The concepts and techniques discussed in the case of the square wave are, nevertheless, universally applicable to all types of composite signals.

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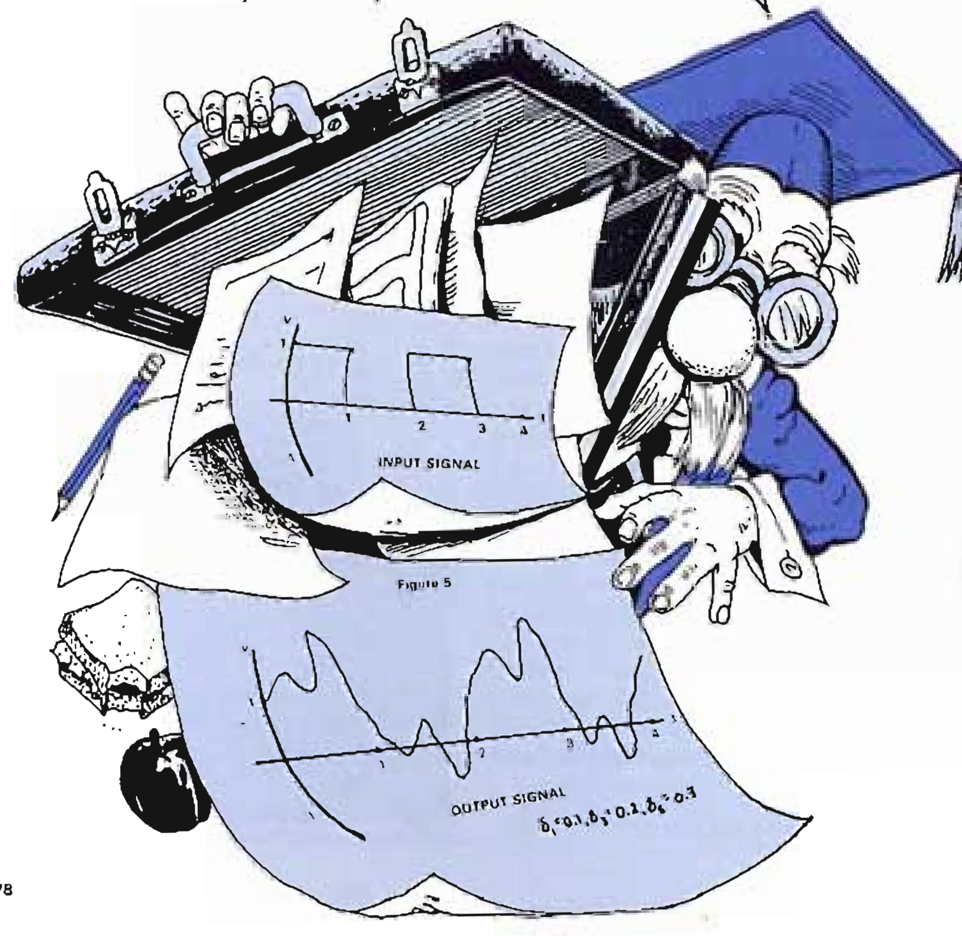

Figure 5